A new pixel-free algorithm of pore-network extraction for fluid flow in porous media: Flashlight search medial axis

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Abstract: Pore-network models have become a critical tool in the study of fluid flow in geo-energy researches over the last few decades, and the accuracy of pore-network modeling results highly depends on the extraction of pore networks. Traditional methods of pore-network extraction are based on pixels and require images with high quality. Here, a pixel-free method called the flashlight search medial axis algorithm is proposed for pore-network extraction in a continuous space. The search domain in a two-dimensional space is a line, whereas a surface domain is searched in a three-dimensional scenario. Thus, the algorithm follows the dimensionality reduction idea; the medial axis can be identified using only a few points instead of calculating every point in the void space. In this way, computational complexity of this method is greatly reduced compared to that of traditional pixel-based extraction methods, thus enabling large-scale pore-network extraction. Based on cases featuring two- and three-dimensional porous media, the algorithm performs well regardless of the topological structure of the pore network or the positions of the pore and throat centers. This algorithm can also be used to examine both closed-boundary and open-boundary cases. Finally, this algorithm can identify the medial axis accurately, which is of great significance in the study of geo-energy.

1. Introduction

Porous media have been considered effective fluid carriers in various geo-energy fields in the last few decades, such as hydrogen and carbon dioxide storage (Makal et al., 2012; Heinemann et al., 2021), and digital rock study in reservoirs (Sun and Zhang, 2020; Shan et al., 2022b). Porous media have complex topological structures induced by various pores and throats (Dullien, 1975; Raoof and Hassanizadeh, 2010), thus, it is imperative that porous media should be accurately described to conduct further numerical investigation.

Porous media are commonly found in a wide range of areas, especially petroleum and natural gas reservoirs (Cao et al., 2020). Advancements in CT scanning have enabled the visualization of the inner void structures of rock matrixes (Mostaghimi et al., 2013; Song et al., 2021), which reveals complex topological structures comprising pores and throats. Problems in porous media have been studied using several numerical approaches, such as the lattice Boltzmann method (Zhang and Sun, 2019), phase field method (Zhu et al., 2020), and smoothed-particle hydrodynamics (Liu et al., 2022a; Feng et al., 2023). Molecular simulation methods have also been employed to reveal the micro-scale mechanisms (Yang et al., 2020; Liu et al., 2024), notably the fluid behavior in nanoscale pores (Liu et al., 2022b; Shan et al., 2022a). However, calculating single- or multi-phase flow in complex pore spaces is usually computationally challenging. Numerical discretization methods, such as finite volume and finite element methods, require the generation of highly accurate unstructured meshes for porous media (Javandel and Witherspoon, 1968; Yang et al., 2019). Therefore, a simplified numerical method is necessary to solve problems in complex porous media.

The first pore-network model (PNM) was proposed by Fatt (1956); a regular two-dimensional lattice was built using

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random radii to predict the capillary pressure and relative permeability. However, a regular pore network is not capable of representing the topology and geometry of porous media. Bryant and Blunt (1992) studied pore-network extraction from a realistic porous media, in which uniform spheres were packed, leading to the conclusion that the pores were packed in a tetrahedral configuration. Furthermore, they predicted the relative permeability of a sand pack, which agreed well with results for sandstone. The growth of PNMs has enabled them to overcome the problems associated with irregular lattices, varying wetting conditions, and multi-phase flow (Blunt et al., 2002; Balhoff and Wheeler, 2009; Ryazanov et al., 2009). This technique is also useful for handling more complex problems, such as non-Newtonian, non-Darcy, and reactive flows (Lopez et al., 2003; Yiotis et al., 2006). Song et al. (2023) proposed a general PNM based on three-phase thermodynamic equilibrium, and predicted multicomponent flow in nanoporous shale. The computational speed of PNMs is considerably faster than that of mesh-based methods; however, their accuracy relies on pore-network extraction.

2. Methodology

2.1 Characteristics of medial axis

The minimum distance between a random point in the void space and a point in the solid phase is determined as follows:

$$\text{dist}(x, D) = \min\{\text{dist}(x, y), y \in D\}$$

where $D$ represents the solid phase, $y$ represents the point in the solid phase, $x$ represents the point in the void phase, and $\text{dist}$ represents the minimum distance. In an image, a pixel of a solid phase can be visualized and considered a point in that solid phase. The distance map is obtained using the above equation. As shown in Fig. 1, the $\text{dist}$ of the pore center is the maximum in that pore space, which is similar to the mountain top. The points on the medial axis are local maxima; the gradient of $\text{dist}$ is discontinued, and the curve resembles a mountain ridge. One special point is the throat center, where $\text{dist}$ is the minimum along the medial axis. The throat center is also a saddle point; the minimum and local maximum are in the direction of the medial axis and the direction perpendicular to the medial axis, respectively. The medial axis comprises the discontinuation points of $\text{dist}$'s gradient, which are called critical points, suggesting that the medial axis can be determined accordingly. As depicted in Fig. 1, the total length between two pores is defined as the...
throat channel is handled as follows: the radii of two inscribed spheres. However, the volume of the
length of the throat channel:
where $L_c$ is the length between the two pore centers, $L_1$ and $L_2$ represent the lengths between the pore centers and the throat center. If the inscribed sphere is considered the pore body, then the following equation is the simplest way to determine the length of the throat channel:

$$L_d = L_1 - R_1 + L_2 - R_2$$

where $L_d$ is the length of the throat channel, $R_1$ and $R_2$ are the radii of two inscribed spheres. However, the volume of the pore body is larger than the volume occupied by the inscribed sphere. The length fraction between the pore body and the throat channel is handled as follows:

$$L_t = L_1 - \alpha \frac{L_1 W_i}{R_1} + L_2 - \alpha \frac{L_2 W_i}{R_2}$$

where $W_i$ is the $dist$ value at the throat center and $\alpha$ is the pore-throat segmentation coefficient. In this regard, the segment coefficient is usually set to 0.5 or 0.6 in previous studies (Oren and Bakke, 2003; Dong and Blunt, 2009).

### 2.2 FSMA pore-network extraction algorithm

#### 2.2.1 Two-dimensional pore-network extraction algorithm

Based on the properties presented in Section 2.1, two-dimensional pore-network extraction is completed as follows:

Step 1: Determine the pore center from a random point in the void space using the steepest-descent method. The calculation domain for the initial point is discretized in the form of a circle to search for the maximum descent. The search range should be as short as possible to ensure the accuracy of pore-center calculation. In our experience, this range should be equivalent to the size of a pixel. Thus, the point is updated with the search step size of the range until the pore center is reached. This process can be likened to a person climbing the steepest route to reach a mountain’s summit.

Step 2: From the first pore center, find the medial axis directions. On the position of pore center, the $dist(x,D)$ is maximum, so there are some critical points lie around the pore center. Circle discretization is applied around the pore center. The medial axis can be determined via a critical point. As seen in Fig. 2, in a case involving three medial axes from one pore center, three critical points are determined using the gradient and Laplace of $dist$.

Step 3: Flashlight searching method to update the critical point on the medial axis. Once the direction of the medial axis is determined, a fan-shaped region is adopted to search for the next critical point. The radio of the fan-shaped region should be smaller than the radio of the circular search region. The critical point can be updated along the medial axis by following this rule. As shown in Fig. 3, the medial axis ($ma$) is determined one step at a time, and the critical point ($cp$) at each step is searched by building a fan-shaped region with a radius of $r_s$. Thus, the total length between two pores ($pc_1$ and $pc_2$) is expressed as follows:

$$L_t = L_1 + L_2 = \sum_{i=1}^{n_1} r_{si} + \sum_{i=1}^{n_2} r_{si}$$

where $r_{si}$ is the search radius at step $i$, $n_1$ is the step number from the initial pore center to the throat center, and $n_2$ is the step number from the throat center to the next pore center.

Step 4: Judge the next pore center and throat center. The critical point is updated via the FSMA method. If the $dist$ on the critical point reaches its maximum value, then that critical point is the pore center. If the $dist$ on the critical point reaches its minimum value, then that critical point is the throat center.

Step 5: Exclude the overlapped pore centers. Since the medial axis will be determined from one pore center to another pore center, there will be some overlapped pore centers. Thus the overlapped pore centers need to be excluded to avoid repetitive calculations.
2.2.2 Three-dimensional pore-network extraction algorithm

The idea underlying three-dimensional pore-network extraction is similar to the above, but the three-dimensional space introduces greater complexity.

Step 1: Find a pore center in the three-dimensional void space by using the steepest-descent method. Unlike the circular search region in the two-dimensional case, a spherical search region is adopted in the three-dimensional study, as seen in Fig. 4(a). The rest of the settings are the same as those in the two-dimensional case.

Step 2: Find the medial axis directions from the first pore center. Three-dimensional discretization is conducted, with the pore center regarded as the sphere center; this is achieved via projection from a cube to a sphere, as shown in Fig. 4(a). The dist map on the sphere surface can then be calculated, as seen in Fig. 4(b). Hence, discretization performs better in terms of mesh homogeneity. In Fig. 4, six local maximum points are observed on the sphere surface, which indicates the presence of six medial axis directions. The cross point of the medial surfaces is the medial axis point, suggesting that several medial surfaces form the medial axis.

Compared with classical (spherical-coordinate) discretization, the projection from the cube to the sphere has better discretization performance. The dist results of both discretization approaches along the medial axis are shown in Fig. 5. Misidentification may occur in Fig. 5(a), as the rough curve may induce some local minimum values along the medial axis. In Fig. 5(b), the dist shows a smoother curve, which is better for identifying the pore and throat centers.

Step 3: Update the critical points on the medial axis via the FSMA algorithm. Unlike the fan-shaped search region in the two-dimensional space, a cone-shaped search region is used in the three-dimensional space. The rest of the settings are the same as those in the two-dimensional case. In this manner, the critical points on the medial axis are determined by identifying the medial surfaces, as shown in Fig. 6.

Step 4: Judge the next pore center and throat centers. In the cone-shaped search region, the medial surface is reduced to a medial axis. Thus, the pore center in the cone-shaped search region is the cross point of several medial axes; the pore center has the maximum dist value. Along the medial axis, composed of the critical points, the throat center can be
Fig. 5. dist values on medial axis obtained via (a) discretization using spherical coordinates and (b) discretization using cubic projection on sphere surface.

**Algorithm 1: FSMA**

**Input**: A random initial point in the void space $p$; the known solid-phase data $s$; the search radius $r_s$ to determine the critical point $c_p$.

**Output**: The pore center $p_c$; the medial axis $ma$.

1. Find the first pore center $p_1$ using the steepest-descent method;

2. for search pore = 1 : m
   // From new-reached pore centers to further-reached pore centers
   do
      3. for search step = 1 : n
         // For the calculation of one strip of medial axis
         do
            4. Discretize the search region;
            5. Determine the neighboring solid data from the previous $c_p$;
            6. Calculate the dist value of discretized points;
            7. Judge the critical point;
               // Introduced in Output and Step 1
               if critical point $c_{p_i}$ reaches the pore center
               // Judge the pore center, in Step 2
               then
                  9. $p_c_i = c_{p_i}$;
                  10. $ma_i = \{c_{p_1}, \ldots, c_{p_i}\}$;
               end
         end
         Update the new-searched pore centers;
         // According to Step 4
   end

Step 5: Exclude the overlapped pore centers. This step is the same as the operation in the two-dimensional scenario.

The pseudocode of the FSMA algorithm is summarized as follows:

### 2.3 Comparative analysis of FSMA and pixel-based algorithms

Regarding to the traditional pore-network extraction methods, such as the maximal ball algorithm, the shape of the inscribed sphere highly depends on the image resolution. If the inscribed sphere is built on a low-resolution image, the boundary information of the inscribed sphere cannot be accurately described, and high-resolution images are computationally expensive. Therefore, the ability of the FSMA, a pixel-free method, to identify the medial axis regardless of image resolution is a natural advantage. Because of the dimensionality reduction idea during the search, computational complexity is reduced compared with that of traditional methods. The computational complexity of the FSMA algorithm is summarized in Table 1. As the steepest-descent method is only used once (for determining the first pore center), its complexity is not considered in the total code. $k$ is the comparison number for the pore and throat centers; this value is usually too small.

**Table 1. Computational complexity of FSMA algorithm.**

<table>
<thead>
<tr>
<th>Line</th>
<th>Complexity</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Line 1</td>
<td>$O(s^2)$</td>
<td>Steepest-descent algorithm</td>
</tr>
<tr>
<td>Lines 6-9</td>
<td>$O(n \log n)$</td>
<td>Search for critical points</td>
</tr>
<tr>
<td>Lines 10-14</td>
<td>$O(k \times n)$</td>
<td>Determination of pore and throat centers</td>
</tr>
<tr>
<td>Lines 16-17</td>
<td>$O(n)$</td>
<td>Updating of pore and throat centers</td>
</tr>
<tr>
<td>Lines 1-18</td>
<td>$O(m \times (n \log n + k \times n + n))$</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>$O(s^2 + m \times (n \log n + k \times n + n)) = O(m n \log n)$</td>
<td></td>
</tr>
</tbody>
</table>

Identified at the minimum of dist.
to add to the computational complexity.

The computational complexity of traditional pore-network extraction methods is expressed as follows:

$$O = (M + N) \left( \frac{L}{\varepsilon} \right)^d$$

where $O$ represents the computational complexity; $M$ and $N$ are the numbers of pore and throat centers; $d$ represents the dimensionality; $L$ represents the size of the computational domain; and $\varepsilon$ is the discretized unit, which is the size of the pixel or voxel in an image. Hence, $L/\varepsilon$ is the number of points that should be calculated. The computational complexity of the FSMA algorithm is expressed as follows:

$$O = (M + N) \log \left( \frac{L}{\varepsilon} \right)$$

In a case where $L/\varepsilon = 10$ in a three-dimensional space, the complexity of the FSMA algorithm is 1,000 times lower than that of traditional methods, in which all points need to be considered for one-time calculation.

3. Validation and discussions

3.1 Pore-network extraction from two-dimensional porous media

Although two-dimensional porous media have few application scenarios, such as artificial micro-fluidic chips, they should still be used for validation. A two-dimensional porous medium is built in a domain sized $100 \times 100$. The solid-phase regions are closed using known boundary points, suggesting that the $\text{dist}$ value in the void phase can be calculated from these solid boundary points. The inner points in the solid phase are unnecessary, which helps save computer memory.

The medial axis starting from the first pore center is identified, as depicted in Fig. 7, which verifies the validity of the FSMA algorithm for the two-dimensional porous medium. On the medial axes, the throat centers can be determined accordingly; in addition, the throat centers are located at the positions of the minimum distance between two spheres, as shown in Fig. 7(a), which enables a clear assessment of the throat center calculation. In Fig. 7(b), the medial axis and th-
process. The center. Moreover, neighbor search is adopted in the search
The pore network is identified gradually, pore center by pore
comprise the topological structure of the porous medium.
axes. As shown in Fig. 8, the pore centers and medial axes
can be searched from the first pore center and its medial
rotor center can also be identified, suggesting that the FSMA
algorithm can search the medial axis from one pore center via
the dimensionality-reduced search method.

Once the medial axis is determined, the other pore centers
can be searched from the first pore center and its medial
axes comprise the topological structure of the porous medium.
The pore network is identified gradually, pore center by pore
center. Moreover, neighbor search is adopted in the search
process. The dist values of the discretized points in the fan-
shaped search region are calculated within the neighbor range,
which is identified from the critical point in the previous
step. Therefore, the pore-network construction of the FSMA
algorithm only needs a small part of the information in the
porous medium.

3.2 Boundary treatment

Boundary treatment is critical for pore-network extraction,
as PNM are used for fluid-flow simulation, which is a
major application field in petroleum engineering (Golparvar
et al., 2018; Zhao et al., 2023). Inflow and outflow boundaries
are necessary for fluid-flow simulation, and dead-end corners
are also considered as special boundaries in porous media
(Lindquist and Venkatarangan, 1999; Yuan et al., 2022). The
FSMA algorithm has a natural advantage in searching dead-
end corners. As the FSMA algorithm is developed by search-
ing medial axes for critical points, a critical point cannot be
found in a dead-end corner within a search region. Therefore,
dead-end corners are determined, as shown in Fig. 9. Despite
the increase in the number of pore centers, the dead-end
center pore can be ignored in some scenarios, such as single-
phase fluid flow. However, in multi-phase fluid-flow problems,
dead-end corners play an important role. Fluids with similar
wettability to the rock surface are trapped in these corners,
thus affecting the subsequent fluid flow.

Solid boundaries are a common boundary type in fluid-flow
simulation, as the connectivity of pores and throats cannot be
confirmed in porous media. In a solid boundary, the surface of
a solid phase and a solid box form new pore channels. Dead-
end corners are also generated in closed solid boxes, and they
can be extracted when necessary. As seen in Fig. 10(a), the
FSMA algorithm can search all critical points and extract the
pore network in solid-box domains; this ability is meaningful
in the study of reservoir rocks which have bad connectivity.

Open boundaries are also examined using the FSMA
algorithm. In cases with open boundaries, the search operation
proceeds until the boundary of the computational domain. The
pore-network extraction is addressed in a domain where the
right and left sides have open boundaries and the pore centers
stop at the boundaries, as shown in Fig. 10(b); these are set as
the Dirichlet boundary condition and the Neumann boundary
condition, respectively (Raeini et al., 2012; Liu et al., 2020),
and the fluid flow can be realized consequently.

3.3 Different packing styles for
three-dimensional space

As shown in Fig. 11, a sphere-packing model is built in a
 cubic box sized 100 × 100 × to verify the FSMA algorithm in a
three-dimensional space. Sixty-four solid spheres are packed
in this box in a 4 × 4 × 4 packing style. The radius of each
sphere is 12.5, suggesting that the spheres are in contact
with each other. The surface of a solid sphere is discretized
through regular hexahedron projection for better computational
performance. The critical points are searched and the medial
axis is identified in the three-dimensional space using the
computational algorithm described in Section 2.2.2.

A regular packing style is adopted because the medial axis
is connected as a regular network and is easily verified.

The beginning state of the medial axis was identified in
Fig. 11(a), which is from the first pore center to a neighbor
pore center through the connection of the medial axis. As
seen in Fig. 11(b), the regular topological structure of the pore
network is then extracted, revealing that the FSMA algorithm
is applicable to three-dimensional porous media. Apart from
the regular model, as depicted in Fig. 12, an irregular sphere-
packing model is used to test the feasibility of the FSMA
algorithm in a more complex porous medium, and it performs
well.

4. Conclusions

In this study, a pore-network extraction algorithm called
FSMA is proposed. The FSMA algorithm is developed in a
continuous space rather than a pixel-based space, which
makes it a pixel-free method. This algorithm adopts the idea
of dimensionality-reduced search, making it unnecessary to
calculate the global data for each pixel point. Thus, the
algorithm has a substantially shorter computational time than
traditional methods. In traditional search procedures, each
point is calculated in the void space, whereas in the FSMA
search procedure, only a few points need to be considered. In
addition, the computation is further accelerated using neighbor
search. Theoretically, the FSMA algorithm extracts pore net-
works with low computational complexity and high efficiency.
These features are highly advantageous in large-scale pore-
network extraction, and this algorithm enables the charac-
terization of the PNM of a reservoir with a representative

Fig. 8. Pore-network extraction from one pore center to other
pore centers.
Fig. 9. Test cases of (a) a straight tube and (b) an elbow tube for FSMA algorithm featuring dead-end corners.

Fig. 10. Pore-network extraction within (a) closed (solid) and (b) open boundaries.

Fig. 11. Three-dimensional regular sphere-packing model and extracted medial axes: (a) beginning state and (b) extended state. Blue line represents the medial axis, and red point represents the pore center.
elementary volume.

Specific computational programs are introduced for two- and three-dimensional spaces, and cases involving two- and three-dimensional porous media are constructed to verify the feasibility of the FSMA algorithm. The results indicate that the topological structure of the pore network is identified and the pore and throat centers are determined accordingly. Furthermore, according to a discussion of the boundary condition for the FSMA algorithm, the algorithm has the natural advantage of being able to search dead-end corners in porous media. Dead-end pore centers are identified by changing the determination conditions of the algorithm for searching critical points, which could be useful for studying multi-phase flow. Additionally, the algorithm performs well in both closed- and open-boundary tests. In conclusion, the pixel-free pore-network extraction algorithm FSMA can extract pore networks at high theoretical computational speeds and therefore has considerable potential for addressing large-scale geo-energy problems.

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Conflict of interest

The authors declare no competing interest.

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